OPTICAL FREQUENCY COMBS AND APPLICATIONS AT NPL

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INTRODUCTION

Optical frequency combs based on mode-locked femtosecond lasers have become widely used tools in optical frequency metrology, providing a convenient and highly accurate way of phase coherently linking optical and microwave frequencies [1, 2]. At NPL we have three such frequency combs, two based on Kerr-lens mode-locked Ti:sapphire lasers with repetition rates of 90 MHz and 800 MHz, and one based on a mode-locked erbium-doped fibre laser with a repetition rate of 100 MHz. These combs are used for a range of applications including optical atomic clocks, low noise microwave synthesis, transmission of ultrastable references over fibre networks and spectroscopy.

In this paper we focus on two specific recent areas of research. The first is the development of a noise-insensitive self-referencing interferometer for carrier-envelope offset frequency stabilization of a Ti:sapphire-based optical frequency comb [3]. The second is the transfer of ultra-low phase noise microwave frequency references over optical fibre networks by transmission of an optical frequency comb centred around 1.56 μm [4].

SELF-REFERENCING SCHEME WITH ENHANCED NOISE IMMUNITY

f-to-2*f* Self-referencing

The ability to measure and stabilize the carrier-envelope offset frequency f_0 of the frequency comb produced by a femtosecond mode-locked laser is key to applications in optical frequency metrology [1, 2] and the use of few-cycle laser pulses to study strong-field phenomena which depend on the precise form of the electric field within each pulse [5]. The most common technique for measuring f_0 is f-to-2f self-referencing [6]. This technique is typically implemented by broadening the laser spectrum using supercontinuum generation in a nonlinear fibre and detecting the beat frequency between the short-wavelength and the frequency-doubled long-wavelength modes of the supercontinuum. The frequency-doubled and fundamental pulses must overlap in time at the detector, so a method of compensating for group-delay dispersion in the nonlinear fibre and frequency doubling crystal is required.

In Ti:sapphire comb systems, group-delay compensation is usually achieved using either a Mach-Zehnder interferometer with an adjustable delay line in one of the arms [6] or a Michelson interferometer [7]. Both topologies are susceptible to acoustic noise, air currents and thermal drift because of the non-common optical path travelled by the two beams. Here we describe a new compact, all-common-path self-referencing scheme which offers tunable group-delay compensation approaching 10 ps, is constructed from off-the-shelf optical components, is easy to align and exhibits significantly lower residual phase noise than a Michelson interferometer arrangement [3].

Experimental Arrangement

The optical design of the new self-referencing interferometer is shown in Fig. 1 and was initially developed for the NPL low repetition rate Ti:sapphire comb system. The Kerr-lens mode-locked Ti:sapphire laser uses an intracavity prism pair for dispersion compensation and produces an average output power of 700 mW with a spectral full width at half maximum of about 30 nm. To achieve an octave spanning spectrum, approximately 30 mW of the laser output is coupled into a 20 cm long microstructured fibre. A combination of a polarizer and a 1064 nm half wave plate at the output of the microstructured fibre ensures that the 532 nm and 1064 nm light is orthogonally polarized at the input of the periodically poled $KTiOPO_4$ (PPKTP) frequency doubling crystal, which is 5 mm long and phase-matched for Type 0 [o + o → o] frequency doubling at 1064 nm. The fundamental and frequency-doubled 532 nm pulses emerging from the crystal thus propagate collinearly and are orthogonally polarized. The group-delay difference between these pulses was measured by coherent spectral interferometry to be about 4 ps and can be compensated using a pair of Wollaston prisms (WPs), in which p- and s-polarized pulses separate at the interface between two birefringent prisms and

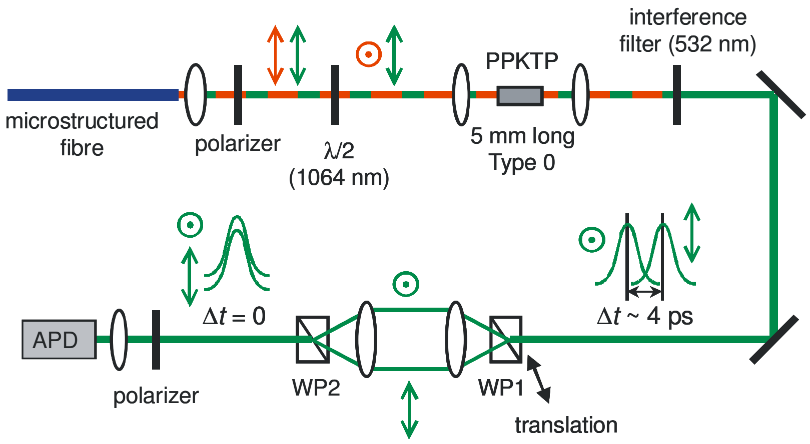

Fig. 1. Schematic of the Wollaston prism interferometer. MSF: microstructured fibre emitting an octave-spanning supercontinuum; P: polarizer; λ/2: 1064 nm half wave plate; WP1, WP2: Wollaston prisms; APD: avalanche photodiode.

experience a differential group-delay whose magnitude and sign depend on the position of the input beam across the aperture of the prism [8]. By adjusting the positions of WP1 and WP2 as shown in Fig. 1, the orthogonally-polarized fundamental and frequency-doubled 532 nm pulses separated by WP1 and recombined by WP2 can be temporally overlapped at the detector. A rotatable polarizer in front of the detector is used to optimize the signal to noise ratio of the beat at frequency f_0.

Phase Noise Performance

To evaluate the residual phase noise performance of this new self-referencing scheme two f-to-$2f$ self-referencing interferometers were constructed. The first was used for generating the error signal used to stabilize f_0 (the in-loop interferometer) and the other was used for making independent phase-noise measurements (the out-of-loop interferometer). To eliminate noise contributions from the microstructured fibre the beams for the two independent interferometers were derived after the fibre by splitting the supercontinuum output using a pellicle beam splitter. Group-delay dispersion compensation was implemented in the in-loop interferometer using a pair of Wollaston prisms. The out-of-loop interferometer was set up in such a way that it could be switched between a Wollaston prism arrangement and a Michelson interferometer using two pairs of flipper mirrors while keeping the frequency-doubling and detection parts of the apparatus common. For all measurements f_0 was stabilized to 30 MHz by a combination of fast feedback to an acousto-optic modulator that controlled the power of the laser pumping the Ti:sapphire laser and slow feedback to a split tubular piezoelectric transducer that adjusted the tilt of the cavity end mirror in the Ti:sapphire laser. For the phase-noise comparison, the two f_0 signals from the in-loop and out-of-loop interferometers were filtered, amplified to a power of 7 dBm and sent to the two inputs of a phase detector. All measurements were performed on an open optical table without any shielding around the interferometers.

In a first set of measurements the voltage fluctuations at the output of the phase detector were recorded in the time domain using a precision digital voltmeter (sampling rate 100 Hz) and the corresponding phase error was calculated as a function of observation time (Fig. 2a). The Wollaston prism interferometer demonstrated significantly better short-term performance, with a root mean square phase error of 37 mrad measured for a 5 s observation time compared with 130 mrad measured for the Michelson interferometer. Longer-term phase drifts were also observed to be lower for the Wollaston prism interferometer.

In a second set of measurements (Fig. 2b) the power spectral density of the phase fluctuations S_ϕ was derived from the voltage fluctuations at the output of the phase detector using a Fast Fourier Transform analyser. The common optical path provided by the Wollaston prism system led to substantial common-mode rejection of acoustic noise in the frequency range from 40 Hz – 1000 Hz, which was observed as a phase noise reduction of up to 15 dB compared with the Michelson interferometer. The sensitivity of the two interferometers to air currents can be inferred from the phase noise observed at frequencies lower than a few hertz, where the Wollaston prism scheme again demonstrated up to 15 dB improvement over the Michelson interferometer. The superior performance of the Wollaston prism interferometer is similarly apparent from a comparison of the magnitude of the steps in the integrated phase noise plots (Fig. 2b).

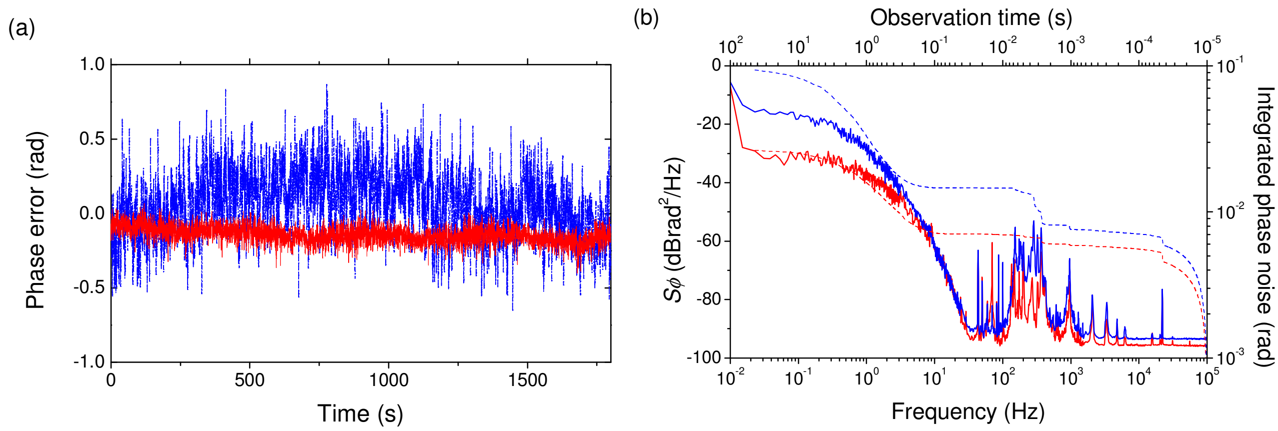

Fig. 2. (a) Phase error between the f_0 beat signals from the in-loop and out-of-loop interferometers as a function of observation time. In both cases the in-loop interferometer uses Wollaston prisms for group-delay dispersion compensation while the out-of-loop interferometer is either a Wollaston prism (red) or Michelson interferometer (blue) system. (b) Comparison of the out-of-loop phase noise S_ϕ (solid lines) and integrated phase noise (dashed lines) measured for the f_0 signals from the Wollaston prism interferometer (red) and Michelson interferometer (blue).

Simplified WP Interferometer

In a further simplification to the interferometer design shown in Fig. 1 we are now using a single-WP interferometer in which the two beams exiting WP1 are directed onto a concave mirror and retro-reflected through WP1. With the concave mirror tilted slightly, the recombined output beam emerges just below the incident beam and can be directed onto the detector. The signal-to-noise ratio in the f_0 beat obtained in this configuration, without the pellicle beam splitter after the microstructured fibre, is 48 dB in a 250 kHz resolution bandwidth. This single-Wollaston-prism design is simpler to align than the two-Wollaston-prism version and provides a compact, low-noise apparatus for detecting f_0. This type of interferometer may prove particularly useful for stabilizing amplified femtosecond laser systems used to study nonlinear processes such as attosecond pulse generation, for which interferometer noise has been reported to be a major contributor to the carrier-envelope phase drift [9].

HIGH STABILITY MICROWAVE FREQUENCY TRANSFER

Introduction

A number of laboratories have demonstrated optical and microwave frequency standards with fractional frequency instabilities at the few parts in 10^{15} level or better for averaging times longer than a few seconds [10]. At present, frequency comparison between atomic frequency standards in distantly located laboratories is usually achieved using satellite-based techniques such as two-way satellite time and frequency transfer or GPS carrier phase observations [11]. However these are unable to preserve the stability delivered by state-of-the-art frequency standards. A promising alternative is to use optical fibre-based transfer techniques. Two principal methods have been explored to date: microwave frequency transfer by amplitude modulation of an optical carrier frequency [12, 13] and optical frequency transfer by direct transmission of the optical carrier itself [12, 14, 15]. The best transfer stability achieved using these techniques is, respectively, 1.3×10^{-15} at 1 s over a distance of 86 km [13] and better than a few parts in 10^{16} at 1 s over fibre lengths spanning from 86 km to 251 km [14–17].

A third technique, whereby optical and microwave references are transferred at the same time by propagation of an optical frequency comb, has been investigated by only one other group. In their work, a microwave transfer stability of better than 9×10^{-15} $(\tau/\mathrm{s})^{-1/2}$ was demonstrated by transmitting an optical frequency comb with a bandwidth of 5 nm – 15 nm over a 6.9 km round-trip installed fibre link, using separate fibres for the forward and return paths [18].

However, the transfer stability was measured at the transmitter end, after the comb had travelled a round trip. Insufficient correlation between the fibre-induced phase changes in the forward and return paths can degrade the actual stability delivered at the "user" end. Here we describe experiments in which an optical frequency comb with a bandwidth of 90 nm is transmitted over a considerably longer round-trip distance of 100 km and an improved microwave transfer stability is measured at the "user" end of the fibre [4].

Experimental Arrangement

The set-up for the frequency transfer experiments is shown in Fig. 3. The output beam of a commercial 1.56 µm amplified erbium-doped mode-locked fibre laser, which emits sub-150 fs optical pulses at a repetition rate of 100 MHz, is split into two optical paths using a 90:10 splitter. The higher power beam is transmitted to the "user" via 50 km of spooled single mode fibre (SMF-28). At the "user" end of the fibre another 90:10 splitter returns the larger fraction of the optical power to the "transmitter" end via a second 50 km fibre spool. This dual-fibre arrangement replicates a typical fibre network, where installed unidirectional optical amplifiers would typically prevent the same fibre being used for the forward and return optical paths. Dispersion-compensating fibre (DCF) modules are used in each 50 km arm to compensate for the chromatic dispersion of the SMF-28 fibre, which would otherwise broaden the laser pulses to about 150 ns after 100 km. With the DCF modules installed, a pulse duration of less than 220 ps is recovered at the "user" end, which for a launched average optical power of approximately 65 mW is sufficient to detect the repetition rate with a signal to noise ratio of greater than 80 dB (resolution bandwidth 1 kHz) after both 50 and 100 km of fibre. The repetition rate of the mode-locked laser is phase-locked to a hydrogen-maser-referenced radiofrequency (RF) synthesizer with a locking bandwidth of a few hundred hertz.

The low power output of the 90:10 power splitter at the "transmitter" end provides the low noise reference against which the round-trip phase noise introduced by the 100 km fibre link is measured. The laser repetition rate is detected here (local repetition rate), after a round trip (returned repetition rate) and at the "user" end (remote repetition rate) using battery-powered high-speed GaAs photodetectors. In each case the 15th harmonic of the repetition rate (1.5 GHz) is selected with a narrow tunable bandpass filter that suppresses the nearest harmonics by more than 30 dB and then amplified to reach efficient driving levels for the subsequent microwave devices.

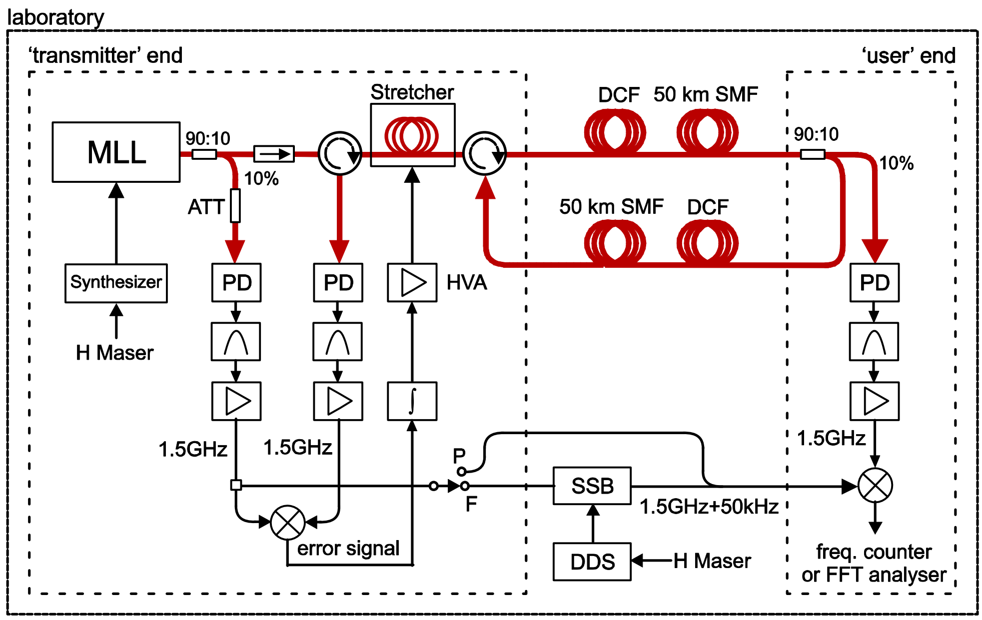

Fig. 3. Block diagram of the experimental setup for transfer of ultra-low phase noise microwave frequencies over 50 km of optical fibre. The measurement system can easily be switched between phase noise (P) and frequency stability (F) measurements of the repetition rate signal delivered to the "user". MLL: mode-locked laser; PD: photodetector; DCF: dispersion-compensating fibre; SSB: single-sideband modulator; HVA: high voltage amplifier; H maser: hydrogen maser; DDS: direct digital synthesizer.

The noise introduced in the fibre link, which arises mainly from mechanical and thermal perturbations to the optical path length, is measured by comparing the local and returned repetition rate signals with a microwave mixer and suppressed using a fibre stretcher. The fibre stretcher was built in-house by winding 250 turns of standard SMF-28 fibre onto a 5 cm diameter piezo tube and can be used to compensate for changes of up to 1.7 mm in the optical path length. Two optical circulators placed before and after the fibre stretcher were used to separate the pulses travelling in the forward and backward directions. With no optical amplifiers along the optical link, it is crucial to achieve low levels of reflected power at the input of the returned repetition rate detector; this is achieved in our setup by fusion-splicing each end of the fibre stretcher to the optical circulators and by using FC/APC connectors everywhere else.

The mechanical and thermal stability of the microwave electronics is also critical. In particular, insufficient thermal stability of the microwave mixer used to construct the error signal for phase noise cancellation can result in increased transfer instability beyond a few seconds, with any unwanted variations of the mixer output voltage being converted directly into a phase error on the extracted repetition rate. For this reason, all the microwave components are rigidly mounted onto aluminium breadboards. The optical power splitters, isolator and circulators, which also show non-negligible sensitivity to changes in ambient temperature, are enclosed in a box lined with thermal foam sheets.

The phase noise and frequency stability of the repetition rate delivered to the "user" are measured with a minimal change to the setup. For the phase noise measurement, the remote repetition rate is phase compared with the local repetition rate using a microwave mixer and processed with a fast Fourier transform analyzer. To measure the fractional frequency stability, a small frequency offset is added to the 15th harmonic of the local repetition rate by inserting a single sideband modulator (SSB) driven by a direct digital synthesizer (DDS), whose clock is referenced to a hydrogen maser. The residual noise floor of the SSB is measured to be 8×10^{-16} $(\tau/\mathrm{s})^{-1}$. An offset frequency of 50 kHz is chosen as this falls into the range where the Λ-type Agilent 53132A frequency counter [19] used in this work shows the best resolution.

Results

The phase noise measured at the "user" end with and without active suppression of the noise introduced in the optical link is shown in Fig. 4. At 1 Hz offset from the carrier the noise is suppressed by more than 10 dB to −101 dBc/Hz. The feedback bandwidth achieved is limited to about 10 Hz by the high voltage amplifier used to drive the fibre stretcher. With improved hardware the feedback bandwidth could be increased towards the theoretical limit of 2 kHz, imposed by the 0.5 ms round-trip travel time of the pulse train.

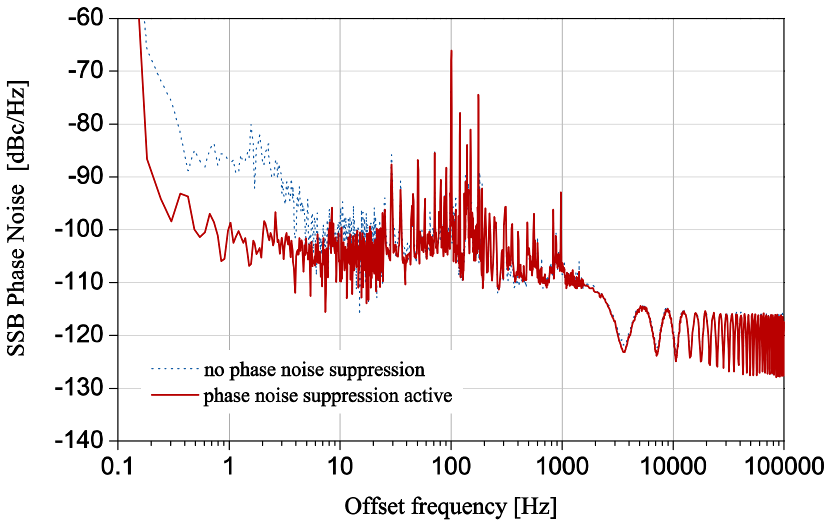

Fig. 4. Phase noise of the 15$^{\mathrm{th}}$ harmonic of the repetition rate delivered at the "user" end of the fibre with and without phase noise suppression. The dips in the phase noise spectrum at frequencies higher than a few kHz are due to self-homodyne detection of the laser noise. Their frequencies correspond to multiples of the inverse of the pulse propagation time over 50 km of fibre.

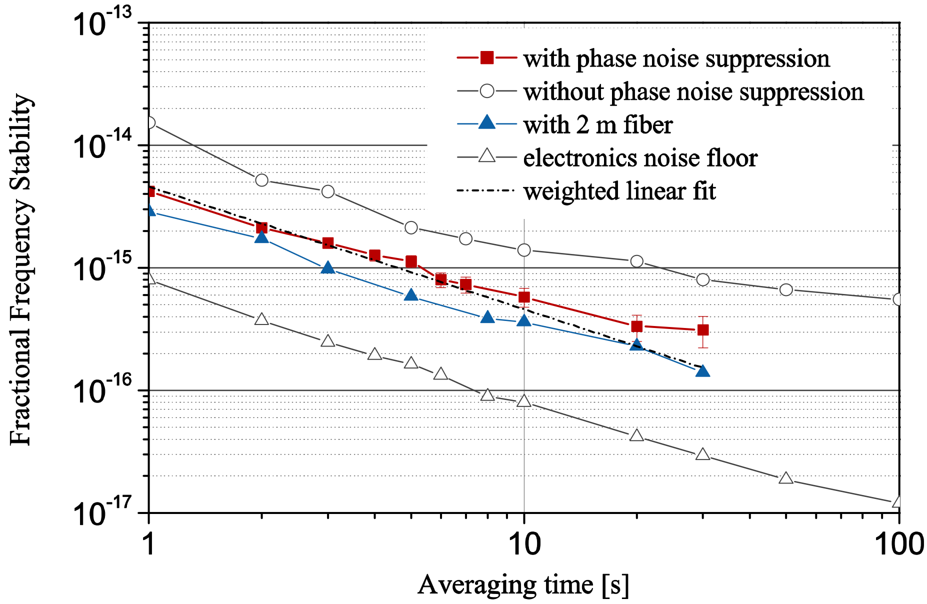

Fig. 5. Fractional frequency stability of the 15th harmonic of the repetition rate delivered at the "user" end of the fibre with (red squares) and without (black circles) phase noise suppression. The optical noise floor (filled blue triangles) and the noise floor imposed by the electronics (open black triangles) are also shown.

The transfer frequency stability measured at the "user" end is shown in Fig. 5. Each data point is obtained from independent frequency stability measurements at different counter gate times, with the measurement time limited to around 250 s by the range of the fibre stretcher. Improved compensation of thermally induced optical path length changes could be achieved in future work by means of a thermally-controlled fibre spool inserted in series with the fibre stretcher. A weighted linear fit to the data, with the slope constrained to be −1, gives a measured transfer instability of 4.6×10^{-15} $(\tau/\mathrm{s})^{-1}$, demonstrating the suitability of this technique for transferring the stability of most high quality microwave or optical references. The measurement noise floor due to the optical to microwave conversion is measured by replacing the 50 km fibre spools with 2 m fibres and inserting optical attenuators to keep the detected signal levels the same. The residual instability due to the microwave electronics following the photodetectors is measured by mimicking the signals from the photodetectors with low noise frequency synthesizers. The lower instability measured in this case, limited by the SSB, indicates that amplitude-to-phase noise conversion in the photodetectors [20] or the residual amplitude sensitivity of the microwave mixer, or a combination of both effects, could be responsible for the higher optical noise floor. Amplitude noise measurements performed with an RF power detector on the input signal applied to the mixer do indeed show levels 15 dB higher when the 1.5 GHz signal is obtained from the photodetectors rather than the RF synthesizers.

Future Work

In summary, the frequency stability of a microwave reference transferred by propagation of an optical frequency comb over 50 km of spooled optical fibre has been measured directly at the "user" end and is found to be 4.6×10^{-15} $(\tau/\mathrm{s})^{-1}$. The phase-noise compensation system has since been upgraded by replacing the home-built fibre stretcher with two commercial fibre stretchers and a thermally-controlled fibre spool. With these improvements, experiments are currently in progress to test the frequency transfer technique on an installed dark fibre network (the JANET Aurora network in the UK). For fibre spans beyond 50 km, the increased optical attenuation requires optical amplifiers to be inserted. Although an *a priori* estimation of noise levels is difficult, a higher level of fibre-induced phase noise is expected in the less protected environment of an installed network. This can be counteracted by increasing the phase noise suppression bandwidth closer to the theoretical limit imposed by the round-trip delay.

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